

Section 11.10 Solutions

math 31

$$\begin{aligned}
 6. \quad f(x) &= \ln(1+x) & f(0) &= 0 \\
 f'(x) &= (1+x)^{-1} & f'(0) &= 1 \\
 f''(x) &= -(1+x)^{-2} & f''(0) &= -1 \\
 f'''(x) &= 2(1+x)^{-3} & f'''(0) &= 2 \\
 f^{(4)}(x) &= -3 \cdot 2(1+x)^{-4} & f^{(4)}(0) &= -3! \\
 f^{(5)}(x) &= 4 \cdot 3 \cdot 2(1+x)^{-5} & f^{(5)}(0) &= 4!
 \end{aligned}$$

$$\begin{aligned}
 \ln(1+x) &= 0 + x - \frac{x^2}{2!} + \frac{2x^3}{3!} - \frac{3!x^4}{4!} + \frac{4!x^5}{5!} - \dots \\
 &= \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} \quad R=1
 \end{aligned}$$

$$\begin{aligned}
 8. \quad f(x) &= e^{-2x} & f(0) &= 1 \\
 f'(x) &= -2e^{-2x} & f'(0) &= -2 \\
 f''(x) &= 4e^{-2x} & f''(0) &= 2^2 \\
 f'''(x) &= -8e^{-2x} & f'''(0) &= -2^3
 \end{aligned}$$

$$\begin{aligned}
 e^{-2x} &= 1 - 2x + \frac{2^2 x^2}{2!} - \frac{2^3 x^3}{3!} + \dots \\
 &= \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{n!} \quad R = \infty
 \end{aligned}$$

$$30. \quad \cos\left(\frac{\pi}{2}x\right) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{2}x\right)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n} x^{2n}}{4^n (2n)!}$$

$$34. \quad x^2 \ln(1+x^3) = x^2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (x^3)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{3n+2}}{n}$$