Section 11.10 Solutions

6. $f(x) = \ln(1+x)$ $f(0) = 0$
$f'(x) = (1+x)^{-1}$ $f'(0) = 1$
$f''(x) = -(1+x)^{-2}$ $f''(0) = -1$
$f'''(x) = 2(1+x)^{-3}$ $f'''(0) = 2$
$f^{(4)}(x) = -3\cdot 2(1+x)^{-4}$ $f^{(4)}(0) = -3!$
$f^{(5)}(x) = 4\cdot 3\cdot 2(1+x)^{-5}$ $f^{(5)}(0) = 4!$

$$\ln(1+x) = 0 + x - \frac{x^2}{2!} + \frac{2x^3}{3!} - \frac{3!x^4}{4!} + \frac{4!x^5}{5!} - \cdots$$
$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n} \qquad R = 1$$

8. $f(x) = e^{-2x}$ $f(0) = 1$
$f'(x) = -2e^{-2x}$ $f'(0) = -2$
$f''(x) = 4e^{-2x}$ $f''(0) = 2^2$
$f'''(x) = -8e^{-2x}$ $f'''(0) = -2^3$

$$e^{-2x} = 1 - 2x + \frac{2^2x^2}{2!} - \frac{2^3x^3}{3!} + \cdots$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{n!}, \qquad R = \infty$$

30. $\cos\left(\frac{\pi}{2}x\right) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{2}x\right)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n} x^{2n}}{4^n (2n)!}$

34. $x^2 \ln(1+x^3) = x^2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x^3)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{3n+2}}{n}$